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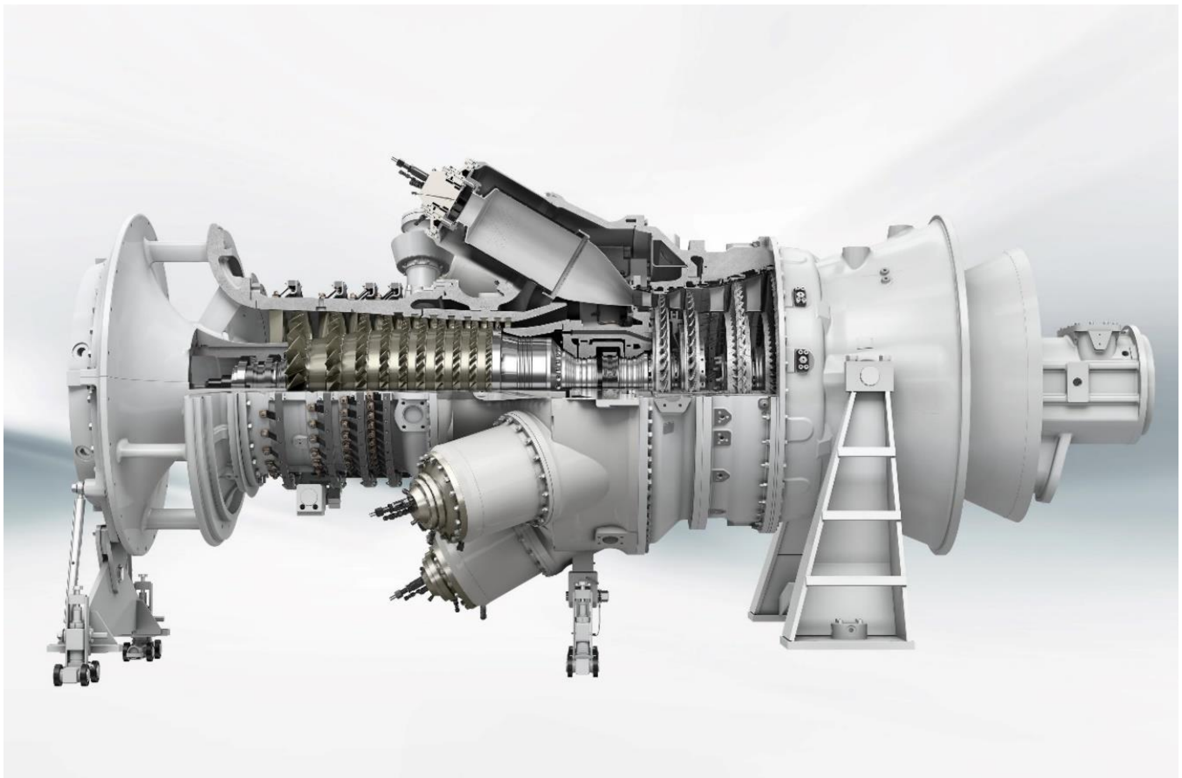

MULTISTAGE GAS TURBINE DESIGN

ME I3100 Steam and Gas Turbine

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ABSTRACT

The two major application areas of gas-turbine engines are aircraft propulsion and electric power generation. Most power-producing devices operate on cycles, and the study of power cycles is an exciting and important part to understand aero-thermodynamics of turbomachinery.

The Brayton cycle is an ideal cycle for a gas turbine cycle. This project discusses the basic design of a gas turbine based on real cycle parameter analysis and with regeneration for efficiency optimization. The project is divided into two parts: First, we perform the basic cycle calculation by obtaining thermodynamic properties at each state. This include enthalpy, temperature, pressure & efficiency. Then the efficiency was optimized by adding regeneration or heat recover technique to increase the total efficiency. Second, aerodynamics and blade parameters were calculated based on the turbine work and other design parameters.

The turbine work was calculated in terms of enthalpy change between the turbine inlet & outlet and this value was used to allocated the number of stages for our design. We also define the reaction for each stage and consequently obtain the efficiency of each stage. The stage efficiencies were in the range of 90-93% which is very high compare to the actual cycle efficiency of 38%. It was observed that the reaction value had significant influence on the blade design, however we applied the mid passage principle in the blade design and conducted an iterative selection of alpha from a specified range which we used in obtaining relative velocities. We further used the Zweifel relation to determine the number of blades for our design and the mass flow rate continuity equation to obtain the blade length.

Finally, we evaluate our design analysis for consistency with defined standards for a gas turbine. We checked parameters such as loading factor and efficiency for each stage which were all satisfied.

NOMENCLATURE

p — Pressure (Psi)
T — Temperature (°R)
h — Enthalpy (Btu/lbm)
q_{in} – Heat Addition (Btu/lbm)
q_{out} – Heat Rejection (Btu/lbm)
w — Work (Btu/lbm)
$\dot{m}$ — Mass Flow Rate (lbm/s)
P — Power (Kw)
N — No of Cycles (Rpm)
N_b – No of Blades
C — Absolute Velocity (ft/s)
W — Relative Velocity (ft/s)
U — Axial Velocity (ft/s)
g_c – Mass — Force constant (lbft/lbf — s^2)
r_p – Pressure Ratio
P_r – Relatve Pressure
η — Efficiency
l — Length of Blade (ft)
r_m – Mid Radius (ft)
r_t – Tip Radius (ft)
r_h – Hub Radius (ft)
ρ — Density (lbm/ft^3)
φ — Loading Coeffient
$\emptyset$ — Zweifel Factor

PART I

CYCLE DESIGN AND CALCULATION

1. BRAYTON CYCLE: THE IDEAL CYCLE FOR GAS-TURBINE ENGINES

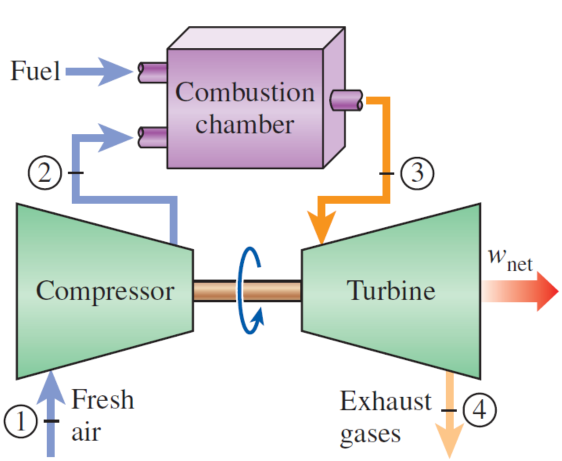


Figure 1 An open-cycle gas-turbine engine

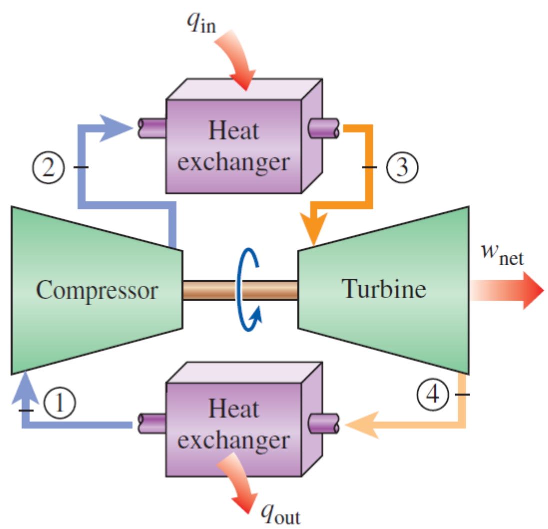
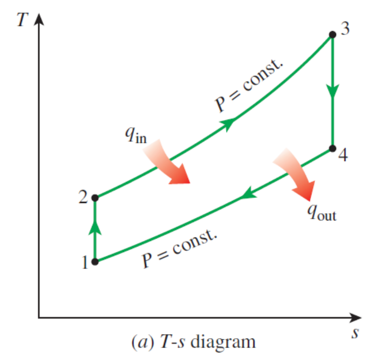


Figure 2 A closed-cycle gas-turbine engine

The ideal cycle that the working fluid undergoes in this closed loop is the Brayton cycle, which is made up of four internally reversible processes:



1-2 Isentropic compression (in a compressor)

2-3 Constant-pressure heat addition

3-4 Isentropic expansion (in a turbine)

4-1 Constant-pressure heat rejection

When the changes in kinetic and potential energies are neglected, the energy balance for a steady-flow process can be expressed, on a unit–mass basis, as

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

Therefore, heat transfers to and from the working fluid are

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

and

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

Then the thermal efficiency of the ideal Brayton cycle under the cold-air standard assumptions becomes

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)}$$

Processes 1-2 and 3-4 are isentropic, and $P_2 = P_3$ and $P_4 = P_1$. Thus

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$$

Substituting these equations into the thermal efficiency relation and simplifying give

$$\eta_{th,Brayton} = 1 - \frac{1}{(r_p)^{\frac{k-1}{k}}}$$

Where, $r_p = \frac{P_2}{P_1}$, is the **pressure ratio** and k is the **specific heat ratio.** Shows that under the cold-air-standard assumptions, the thermal efficiency of an ideal Brayton cycle depends on the pressure ratio of the gas turbine and the specific heat ratio of the working fluid. The thermal efficiency increases with both of these parameters, which is also the case for actual gas turbines.

Given,

Net Shaft Power = 10,000.00 [HP]

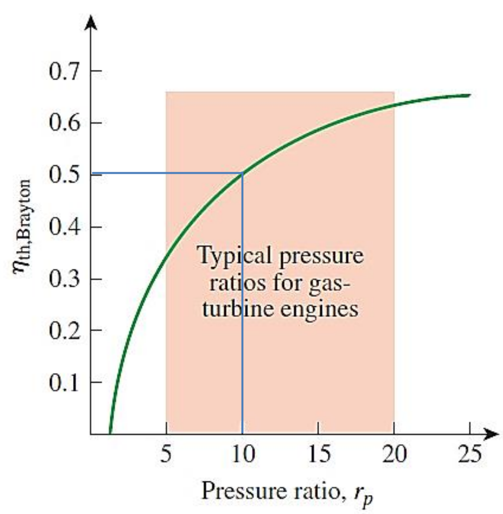
Turbine Inlet temperature (T_a) = 1800 [F]

Design RPM = 7200

Assumptions:

Air is an ideal gas $(k = 1.4)$

Isentropic expansion and compression



2. Design of Ideal Gas-Turbine Cycles

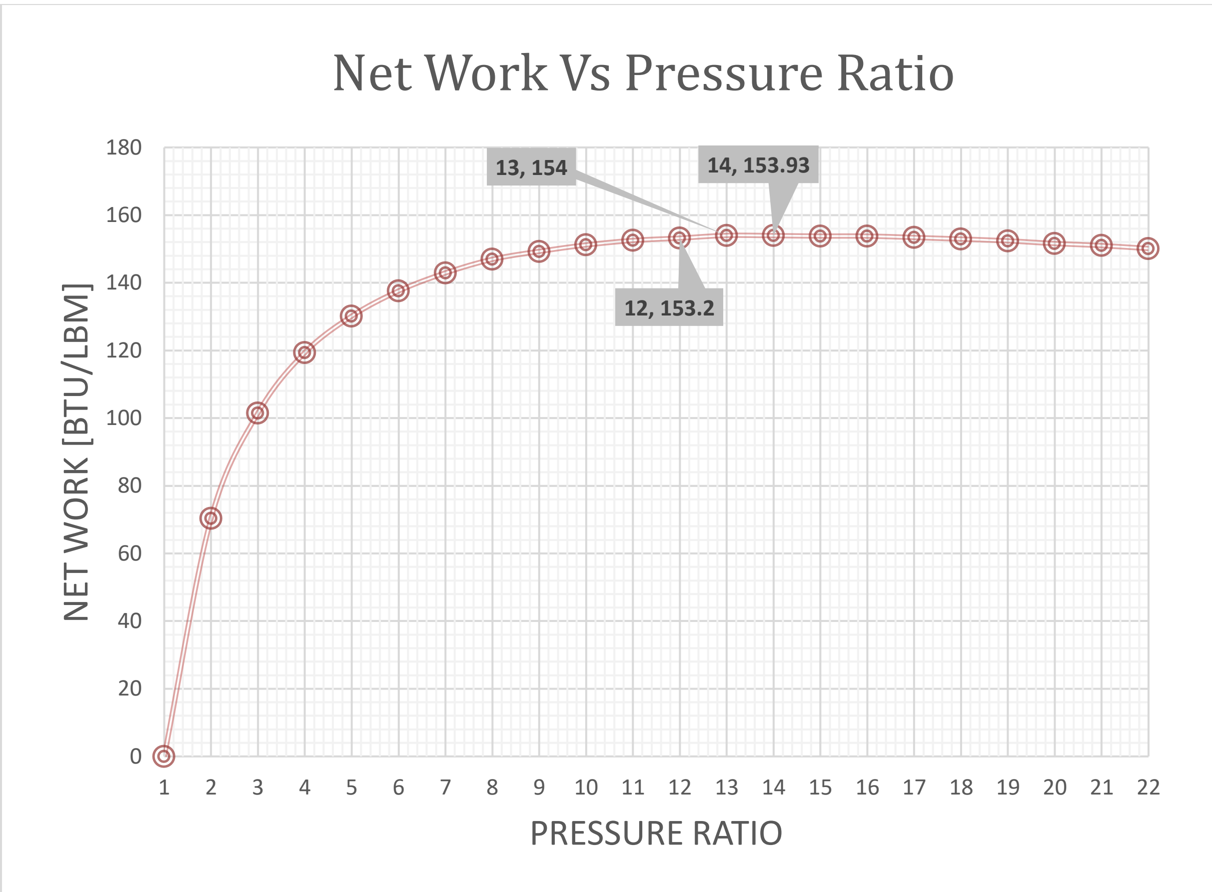
Step 1: Process 1–2 (isentropic compression of an ideal gas):

$T_1 = 100\ [F]$ (Reference temperature)

$$T_1(R) = T(°\mathrm{F}) + 459.67 = 70 + 459.67 = 529.67\ [R]$$

$$h_1\ (T_1) = 126.45\ \left[\frac{Btu}{lbm}\right]$$

$$P_{r1}(T_1) = 1.297$$



$P_{r2} = r_p * P_{r1} = 13 * 1.5742 = 16.860$

$$T_2(P_{r2}) = 1090.01\ [R]$$

$$T_2@\ max_{net\ work} = \sqrt{T_1 T_3} = \sqrt{529.67 * 2259.67} = 1094.01\ [R]$$

$$T_2(P_{r2}) \cong T_2@\ max_{net\ work}$$

$$h_2(T_2) = 263.49\ \left[\frac{Btu}{lbm}\right]$$

Step 2: Process 3–4 (isentropic expansion of an ideal gas):

$T_3 = 1800\ [F]$

$$T_3(R) = T(°\text{F}) + 459.67 = 1800 + 459.67 = 2259.67\ [R]$$

$$h_3\ (T_3) = 577.42\ \left[\frac{Btu}{lbm}\right]$$

$$P_{r3}(T_3) = 286.56$$

$$P_{r4} = \frac{1}{r_p} * P_{r3} = \frac{286.56}{13} = 22.04$$

$$T_4(P_{r4}) = 1172.20\ [R]$$

$$h_4(T_4) = 284.24\ \left[\frac{Btu}{lbm}\right]$$

Step 3: To find the back work ratio, we need to find the work input to the compressor and the work output of the turbine:

$$w_{turbine} = h_3 - h_4 = (577.42 - 284.24)\left[\frac{Btu}{lbm}\right] = \mathbf{293.18}\left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$w_{comp} = h_2 - h_1 = (263.49 - 126.45)\left[\frac{Btu}{lbm}\right] = \mathbf{137.04}\left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$w_{net} = w_{turbine} - w_{comp} = (293.18 - 137.04)\left[\frac{Btu}{lbm}\right] = \mathbf{156.14}\left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$r_{bw} = \frac{w_{comp}}{w_{turbine}} = \frac{137.04}{293.18} = 0.4674$$

That is, **46.74%** percent of the turbine work output is used just to drive the compressor.

Step 4: The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

$$q_{in} = h_3 - h_2 = (577.42 - 263.49)\left[\frac{Btu}{lbm}\right] = 313.93\left[\frac{Btu}{lbm}\right]$$

$$q_{out} = h_4 - h_1 = (284.24 - 126.45)\left[\frac{Btu}{lbm}\right] = 157.79\left[\frac{Btu}{lbm}\right]$$

$$\eta_{th,Brayton} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{157.79}{313.93} = \mathbf{49.73\%}$$

Step 5: Mass flow Rate:

$$Power_{net} = 10{,}000\ [hp] = 7070\left[\frac{Btu}{s}\right]$$

$$Mass\ Flow\ Rate\ (\dot{m}) = \frac{Power_{net}}{w_{net}}\left[\frac{\frac{Btu}{s}}{\frac{Btu}{lbm}}\right] = \frac{7070}{156.14}\left[\frac{\boldsymbol{lbm}}{\boldsymbol{s}}\right] = \mathbf{45.28}\left[\frac{\boldsymbol{lbm}}{\boldsymbol{s}}\right]$$

3. Deviation of Actual Gas-Turbine Cycles from Idealized Ones

The actual work input to the compressor is more, and the actual work output from the turbine is less because of irreversibilities. The deviation of actual compressor and turbine behavior from the idealized isentropic behavior can be accurately accounted for by utilizing the isentropic efficiencies of the turbine and compressor as

$$\eta_c = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

and

$$\eta_t = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

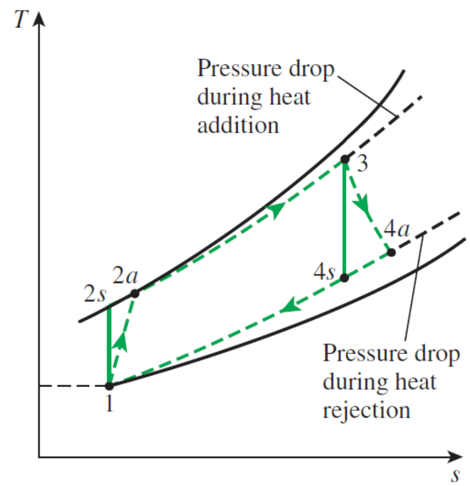


Figure 3 The deviation of an actual gas-turbine

where states 2a and 4a are the actual exit states of the compressor and the turbine, respectively, and 2s and 4s are the corresponding states for the isentropic case, as illustrated in Figure 3.

For specified turbine and compressor efficiencies,

$$\eta_{comp} = \mathbf{90\%}\ (assume)$$

$$\eta_{turbine} = \mathbf{85\%}\ (assume)$$

Step 6: Actual Work

$$w_{comp_a} = \frac{w_{comp_s}}{\eta_{comp}} = \frac{h_{2_s} - h_1}{0.8} = \frac{(263.49 - 126.45)}{0.9}\left[\frac{Btu}{lbm}\right] = \mathbf{152.26}\left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$$

$$w_{turbine_a} = \eta_{turbine} w_{turbine_s} = 0.85 * \left(h_3 - h_{4_s}\right) = 0.85 * (577.42 - 284.24)\left[\frac{Btu}{lbm}\right] = \mathbf{249.20}\left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$$

$$w_{net_a} = w_{turbine_a} - w_{comp_a} = 249.20 - 152.26\ \left[\frac{Btu}{lbm}\right] = \mathbf{96.94}\left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$$

$$r_{bw_a} = \frac{w_{comp_a}}{w_{turbine_a}} = \frac{152.26}{249.20} = 0.6109$$

That is, $\mathbf{61.09\%}$ percent of the turbine work output is used just to drive the compressor.

Step 7: Actually Enthalpy

$$h_{2_a} = h_1 + w_{comp_a} = 126.45 + 152.26 \left[\frac{Btu}{lbm}\right] = \mathbf{278.71} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$h_{4_a} = h_3 - w_{turbine_a} = 577.42 - 249.20 \left[\frac{Btu}{lbm}\right] = \mathbf{328.22} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

Step 8: The actual thermal efficiency of the cycle is the ratio of the actual net heat output to the total heat input:

$$q_{in_a} = h_3 - h_{2_a} = 577.42 - 278.71 \left[\frac{Btu}{lbm}\right] = \mathbf{298.71} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

and

$$q_{out_a} = h_{4_a} - h_1 = 328.22 - 126.45 \left[\frac{Btu}{lbm}\right] = \mathbf{201.77} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$\eta_{th,Brayton_a} = 1 - \frac{q_{out_a}}{q_{in_a}} = 1 - \frac{201.77}{298.71} = \mathbf{32.45\%}$$

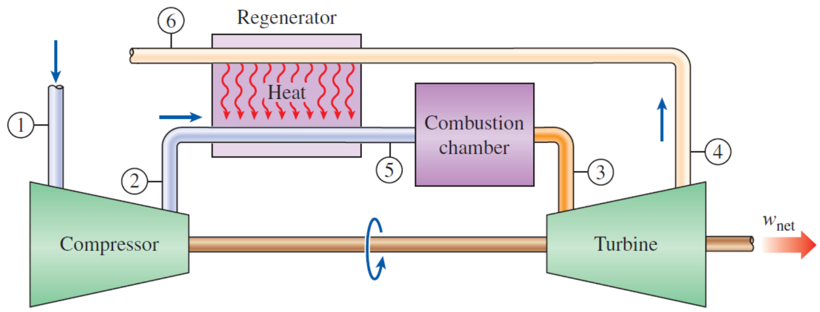
Step 9: Actual Mass flow Rate:

$$Power_{net} = 10{,}000.00\ [hp] = 7070 \left[\frac{Btu}{s}\right]$$

$$Mass\ Flow\ Rate\ (\dot{m}) = \frac{Power_{net}}{w_{net_a}} \left[\frac{\frac{Btu}{s}}{\frac{Btu}{lbm}}\right] = \frac{7070}{96.94} \left[\frac{Btu}{s}\right] = \mathbf{72.94} \left[\frac{\mathbf{lbm}}{\mathbf{s}}\right]$$

4. Actual Brayton Cycle with Regeneration

In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor. Therefore, the high-pressure air leaving the compressor can be heated by transferring heat to it from the hot exhaust gases in a counter-flow heat exchanger, which is also known as a regenerator or a recuperator.



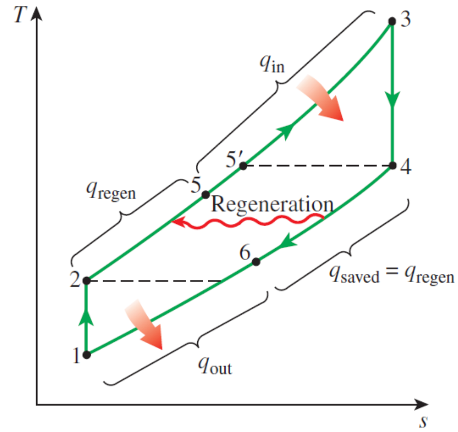


Figure 4 T-s diagram of a Brayton cycle with regeneration

The thermal efficiency of the Brayton cycle increases as a result of regeneration since the portion of energy of the exhaust gases that is normally rejected to the surroundings is now used to preheat the air entering the combustion chamber. This, in turn, decreases the heat input (thus fuel) requirements for the same net work output.

The extent to which a regenerator approaches an ideal regenerator is called the effectiveness ε and is defined as

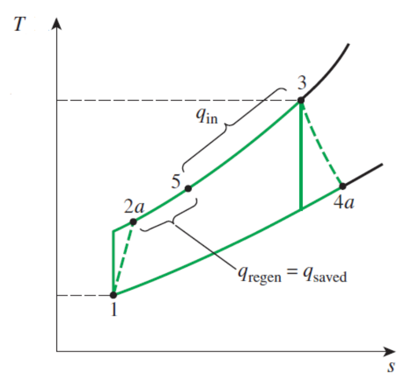
$$\varepsilon = \frac{Q_{reg,actual}}{Q_{reg,max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

Step 10: Actual Heat Addition with Regeneration

$\varepsilon = \ 90\% \ (assume)$

$$\varepsilon = \frac{Q_{reg,actual}}{Q_{reg,max}} = \frac{h_5 - h_{2_a}}{h_{4_a} - h_{2_a}}$$

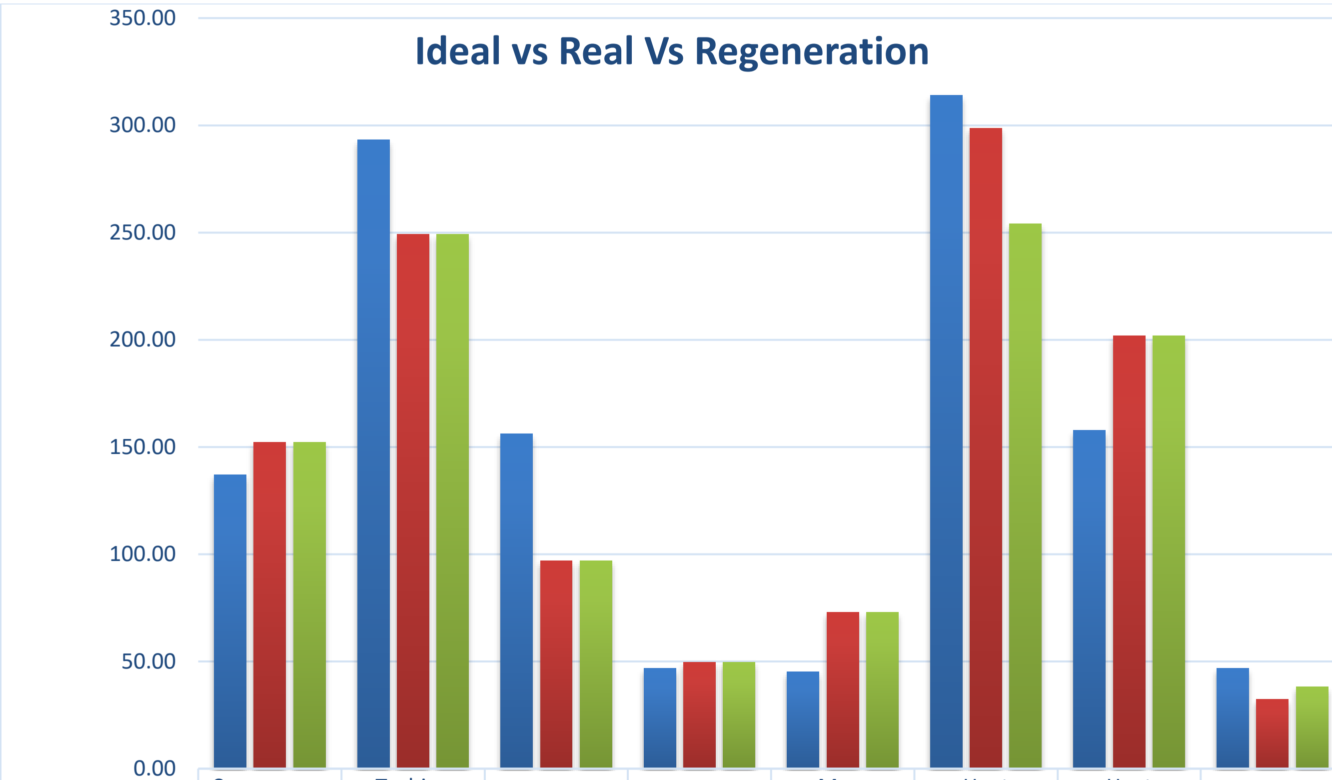
$$h_5 = \varepsilon * \left(h_{4_a} - h_{2_a}\right) + h_{2_a}$$
$$= 0.9 * (328.22 - 278.71) + 278.71 \left[\frac{Btu}{lbm}\right] = \mathbf{323.27} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$



$$q_{in_{a,reg}} = h_3 - h_5 = 577.42 - 323.27 \left[\frac{Btu}{lbm}\right] = \mathbf{254.15} \left[\frac{\mathbf{Btu}}{\mathbf{lbm}}\right]$$

$$\eta_{th,Brayton_a} = \frac{w_{net}}{q_{in_{a,reg}}} = \frac{96.94}{254.15} = \mathbf{38.14\%}$$

Summary of Cycle:



	Compressor Work Input [Btu/lbm]	Turbine Work Output [Btu/lbm]	Net Work [Btu/lbm]	Work back Ratio [%]	Mass Flowrate [lbm/s]	Heat Supplied [Btu/lbm]	Heat Rejected [Btu/lbm]	Thermal Efficiency [%]
Ideal	137.04	293.18	156.15	46.74	45.28	313.94	157.79	46.74
Actual	152.26	249.21	96.94	49.74	72.94	298.71	201.77	32.45
Regeneration	152.26	249.21	96.94	49.74	72.94	254.16	201.77	38.14

The thermal efficiency of the gas turbine has gone up from 32.45 to 38.14 percent as a result of installing a regenerator that helps to recuperate some of the thermal energy of the exhaust gases. However, achieving a higher effectiveness requires the use of a larger regenerator, which carries a higher price tag and causes a larger pressure drop. Therefore, the use of a regenerator with a very high effectiveness cannot be justified economically unless the savings from the fuel costs exceed the additional expenses involved.

PART II

AERODYNAMICS CALCULATION

5. Stage Design

We have,

$$w_{turbine_a} = \mathbf{249.20} \left[\frac{Btu}{lbm}\right]$$

Let's specify, the turbine will be of 3 stage with $\boldsymbol{\Delta h_1} = \mathbf{86.20}\,, \boldsymbol{\Delta h_2} = \mathbf{83.00}\ \boldsymbol{and}\ \boldsymbol{\Delta h_3} = \mathbf{80.00}\ \left[\frac{Btu}{lbm}\right]$

5.1 1st Stage Design

5.1.1 Specify Δh for the 1st stage:

Δh for 1st stage is

We have $\boldsymbol{\Delta h_1} = \mathbf{86.2} \left[\frac{Btu}{lbm}\right]$

5.1.2 Specify Degree of Reaction (R):

Let's specify $\boldsymbol{R_1} = \mathbf{0.3}$

5.1.3 Enthalpy Distribution

Enthalpy in Rotor $\Delta h_r = R_1 * \Delta h_1 = 0.3 * 86.2 = \mathbf{25.86}\ \left[\frac{Btu}{lbm}\right]$

Enthalpy in Stator $\Delta h_s = \Delta h_1 - \Delta h_r = 86.2 - 25.86 = \mathbf{60.34}\ \left[\frac{Btu}{lbm}\right]$

5.1.4 Adiabatic Velocity

Adiabatic velocity $C_1 = \sqrt{2 * g_c * \Delta h_1}$

$$= \sqrt{2 * 32.2\ \left[\frac{lbm.ft}{lbf.s^2}\right] * 86.2 * 778 \left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{2078.20}\ \left[\frac{ft}{s}\right]$$

5.1.5 Velocity Ratio

Velocity ratio $\left(\frac{U}{C_1}\right)$ is a function of R. Using Linear approximation.

$$\frac{U}{C_1} \cong \mathbf{0.6}$$

5.1.6 Blade Tangential Velocity

Blade Tangential Velocity $U = 0.6 * C_1 = 0.707 * 2078.20 = \mathbf{1241.25}\ \left[\frac{ft}{s}\right]$

5.1.7 Speed

Speed $N = 7200\ [RPM] = 7200 * \frac{2\pi}{60}\ \left[\frac{rad}{s}\right] = \mathbf{753.98} \left[\frac{rad}{s}\right]$

5.1.8 Mean Radius

Root Mean Radius $r_m = \frac{U}{N} = \frac{1241.25}{753.98}\left[\frac{ft}{s}\right]\left[\frac{s}{rad}\right] = \mathbf{1.65}[\boldsymbol{ft}]$

5.1.9 Absolute Velocity at Stator Exit

Absolute Velocity $C_2 = \sqrt{2 * g_c * (1 - R_1) * \Delta h_1}$

$$= \sqrt{2 * 32.2 \left[\frac{lbm.ft}{lbf.s^2}\right] * (1 - 0.3) * 86.2 * 778 \left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{1738.74} \left[\frac{ft}{s}\right]$$

5.1.10 Absolute Velocity Angle at Stator Exit

Absolute Velocity Angle α_2 is in the range of (70° - 80°). Let's specify $\alpha_2 = \mathbf{73}°$

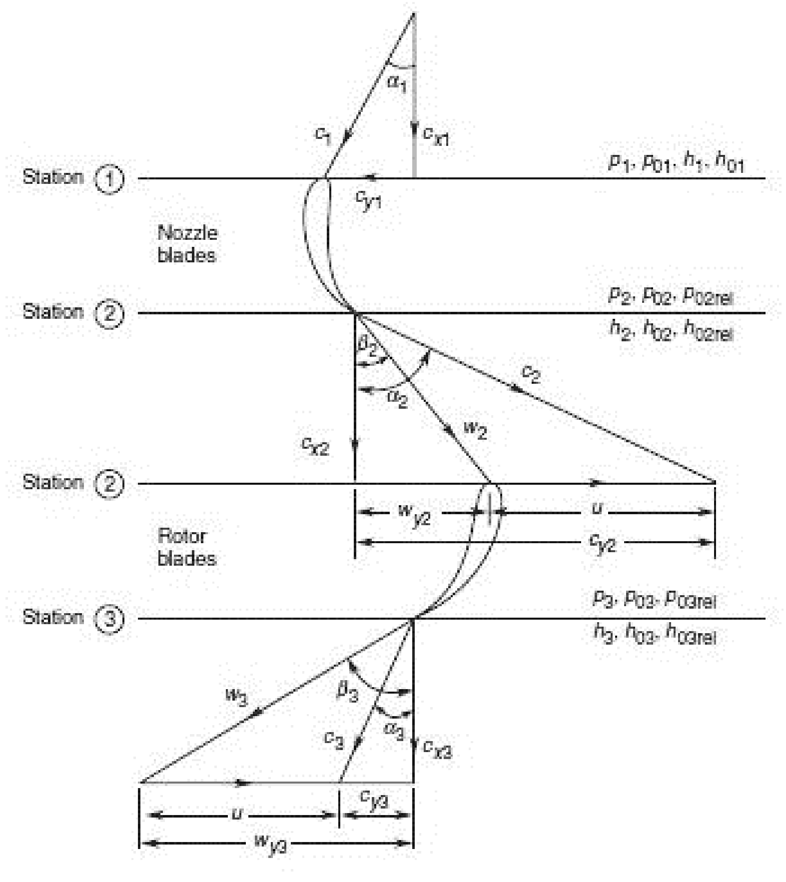


Figure 6 Velocity Diagram

5.1.11 Axial Velocity Component at Stator Exit

Axial Velocity Component $C_{x2} = C_2 \cos \alpha_2$

$$= 1738.74 * \cos 73$$

$$= \mathbf{508.36} \left[\frac{ft}{s}\right]$$

5.1.12 Relative Velocity at Stator Exit

Relative Velocity $W_2 = \sqrt{C_2^2 + U^2 - 2 * C_2 * U * \sin\alpha_2}$

$$= \sqrt{1738.74^2 + 1241.25^2 - 2 * 1738.74 * 1241.25 * \sin 73}$$

$$= \mathbf{660.39}\ \left[\frac{ft}{s}\right]$$

5.1.13 Relative Velocity Angle at Stator Exit

Relative Velocity Angle β_2

We have $W_2 \cos\beta_2 = C_{x2} \rightarrow \beta_2 = \cos^{-1}\left(\frac{C_{x2}}{W_2}\right) = \cos^{-1}\left(\frac{508.36}{660.39}\right) = \mathbf{39.66°}$

5.1.14 Relative Velocity Angle at Rotor Exit

Relative Velocity Angle β_3

We have, $R_1 = \frac{C_{x2}}{U}\left(\frac{\tan\beta_3 - \tan\beta_2}{2}\right)$

$$\beta_3 = \tan^{-1}\left(\frac{2UR}{C_{x2}} + \tan\beta_2\right) = \tan^{-1}\left(\frac{2 * 1241.25 * 0.3}{508.36} + \tan 39.66\right) = \mathbf{66.45°}$$

5.1.15 Axial Velocity Component at Rotor Exit

Axial Velocity Component $C_{x3} = C_{x2} = \mathbf{508.36}\ \left[\frac{ft}{s}\right]$

5.1.16 Relative Velocity at Rotor Exit

We have, $W_3 \cos\beta_3 = C_{x3}$

Relative Velocity $W_3 = \frac{C_{x3}}{\cos\beta_3}$

$$= \frac{508.36}{\cos 66.45} = \mathbf{1272.25}\ \left[\frac{ft}{s}\right]$$

5.1.17 Absolute Velocity at Rotor Exit

Absolute Velocity $C_3 = \sqrt{W_3^2 + U^2 - 2 * W_3 * U * \sin\beta_3}$

$$= \sqrt{1272.25^2 + 1241.25^2 - 2 * 1272.25 * 1241.25 * \sin 66.45}$$

$$= \mathbf{513.86}\ \left[\frac{ft}{s}\right]$$

5.1.18 Absolute Velocity Angle at Rotor Exit

Absolute Velocity Angle α_3 is in the range of (0° - 5°)

We have, $C_3 \cos\alpha_3 = C_{x3} \rightarrow \alpha_3 = \cos^{-1}\left(\frac{C_{x3}}{C_3}\right)$

$$= \cos^{-1}\left(\frac{508.36}{513.86}\right) = \mathbf{8.39°}$$

5.1.19 Work

Work $w = \frac{U}{g_c}(W_2 \cos\beta_2 + W_3 \cos\beta_3)$

$$= \frac{1241.25}{32.2*778}\left[\frac{ft}{s}\right]\left[\frac{lbf.s^2}{lbm.ft}\right]\left[\frac{Btu}{lbf.ft}\right](660.39\ \sin 39.66 + 1272.25\ \sin 66.45)\left[\frac{ft}{s}\right]$$

$$= \mathbf{78.67}\left[\frac{\mathbf{Btu}}{\mathbf{lbm.}}\right]$$

5.1.20 Stage Efficiency

Efficiency $\eta = \frac{2wg_c}{C_1^2}$

$$= \frac{2*78.67*32.2*778}{2078.20^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right]$$

$$= \mathbf{91}\%$$

5.1.21 Length of Blade

Length $l = \frac{\dot{m}}{\rho*2\pi*r_m*C_{x2}} = \frac{72.94}{0.0748*2\pi*1.65*508.36}\left[\frac{lbm}{s}\frac{ft^3}{lbm}\frac{1}{ft}\frac{s}{ft}\right] = \mathbf{0.185}\ [\boldsymbol{ft}]$

5.1.22 Spacing of Blade

Assume $b \approx c = 3$ inch (c is chord length)

Spacing $s = \frac{0.85*b}{2*(\tan\beta_2+\tan\beta_3)\cos^2\beta_3} = \frac{2*0.85*3}{2*12*(\tan 39.66+\tan 66.45)\cos^2 66.45} = \mathbf{0.213}\ [\boldsymbol{ft}]$

5.1.23 Number of Blade

Number $N_b = \frac{2\pi r_m}{s} = \frac{2\pi*1.65}{0.213}\left[\frac{ft}{ft}\right] \approx \mathbf{49}$

5.1.24 Loading Factor

Loading Factor $\psi = \frac{g_c w}{U^2} = \frac{32.2*78.67*778}{1241.25^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right] = \mathbf{1.27} < \mathbf{2}$

5.2 2nd Stage Design

5.2.1 Specify Δh for the 2nd stage:

Δh for 2nd stage is

We have $\boldsymbol{\Delta h_2} = \mathbf{83.0}\left[\frac{Btu}{lbm}\right]$

5.2.2 Specify Degree of Reaction (R):

Let's specify $\boldsymbol{R_2} = \mathbf{0.32}$

5.2.3 Enthalpy Distribution

Enthalpy in Rotor $\Delta h_r = R_2 * \Delta h_2 = 0.32 * 83 = \mathbf{26.56}\ \left[\frac{Btu}{lbm}\right]$

Enthalpy in Stator $\Delta h_s = \Delta h_2 - \Delta h_r = 83 - 26.56 = \mathbf{56.44}\ \left[\frac{Btu}{lbm}\right]$

5.2.4 Adiabatic Velocity

Adiabatic velocity $C_1 = \sqrt{2 * g_c * \Delta h_2}$

$$= \sqrt{2 * 32.2\ \left[\frac{lbm.ft}{lbf.s^2}\right] * 83 * 778\left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{2039.26}\ \left[\frac{ft}{s}\right]$$

5.2.5 Velocity Ratio

Velocity ratio $\left(\frac{U}{C_1}\right)$ is a function of R. Using Linear approximation.

$$\frac{U}{C_1} \cong \mathbf{0.61}$$

5.2.6 Blade Tangential Velocity

Blade Tangential Velocity $U = 0.61 * C_1 = 0.61 * 2039.26 = \mathbf{1236.72}\ \left[\frac{ft}{s}\right]$

5.2.7 Speed

Speed $N = 7200\ [RPM] = 7200 * \frac{2\pi}{60}\ \left[\frac{rad}{s}\right] = \mathbf{753.98}\left[\frac{rad}{s}\right]$

5.2.8 Mean Radius

Root Mean Radius $r_m = \frac{U}{N} = \frac{1236.72}{753.98}\left[\frac{ft}{s}\right]\left[\frac{s}{rad}\right] = \mathbf{1.64}[\boldsymbol{ft}]$

5.2.9 Absolute Velocity at Stator Exit

Absolute Velocity $C_2 = \sqrt{2 * g_c * (1 - R_2) * \Delta h_2}$

$$= \sqrt{2 * 32.2\ \left[\frac{lbm.ft}{lbf.s^2}\right] * (1 - 0.32) * 83 * 778\left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{1681.61}\ \left[\frac{ft}{s}\right]$$

5.2.10 Absolute Velocity Angle at Stator Exit

Absolute Velocity Angle α_2 is in the range of (70° - 80°). Let's specify $\alpha_2 = \mathbf{75°}$

5.2.11 Axial Velocity Component at Stator Exit

Axial Velocity Component $C_{x2} = C_2 \cos \alpha_2$

$$= 1681.61 * \cos 75$$

$$= \mathbf{435.23} \left[\frac{ft}{s}\right]$$

5.2.12 Relative Velocity at Stator Exit

Relative Velocity $W_2 = \sqrt{C_2^2 + U^2 - 2 * C_2 * U * \sin \alpha_2}$

$$= \sqrt{1681.61^2 + 1236.72^2 - 2 * 1681.61 * 1236.72 \ * \sin 75}$$

$$= \mathbf{582.80} \left[\frac{ft}{s}\right]$$

5.2.13 Relative Velocity Angle at Stator Exit

Relative Velocity Angle β_2

We have $W_2 \cos \beta_2 = C_{x2} \rightarrow \beta_2 = \cos^{-1}\left(\frac{C_{x2}}{W_2}\right) = \cos^{-1}\left(\frac{435.23}{582.80}\right) = \mathbf{41.69°}$

5.2.14 Relative Velocity Angle at Rotor Exit

Relative Velocity Angle β_3

We have, $R_2 = \frac{C_{x2}}{U}\left(\frac{\tan \beta_3 - \tan \beta_2}{2}\right)$

$$\beta_3 = \tan^{-1}\left(\frac{2UR_2}{C_{x2}} + \tan \beta_2\right) = \tan^{-1}\left(\frac{2 * 1236.72 * 0.32}{435.23} + \tan 41.69\right) = \mathbf{69.74°}$$

5.2.15 Axial Velocity Component at Rotor Exit

Axial Velocity Component $C_{x3} = C_{x2} = \mathbf{435.23} \left[\frac{ft}{s}\right]$

5.2.16 Relative Velocity at Rotor Exit

We have, $W_3 \cos \beta_3 = C_{x3}$

Relative Velocity $W_3 = \frac{C_{x3}}{\cos \beta_3}$

$$= \frac{435.23}{\cos 69.74} = \mathbf{1256.86} \left[\frac{ft}{s}\right]$$

5.2.17 Absolute Velocity at Rotor Exit

Absolute Velocity $C_3 = \sqrt{W_3^2 + U^2 - 2 * W_3 * U * \sin \beta_3}$

$$= \sqrt{1272.25^2 + 1241.25^2 - 2 * 1272.25 * 1241.25 \ * \sin 66.45}$$

$$= \mathbf{439.03} \left[\frac{ft}{s}\right]$$

5.2.18 Absolute Velocity Angle at Rotor Exit

Absolute Velocity Angle α_3 is in the range of (0° - 5°)

We have, $C_3 \cos \alpha_3 = C_{x3} \rightarrow \alpha_3 = \cos^{-1}\left(\frac{C_{x3}}{C_3}\right)$

$$= \cos^{-1}\left(\frac{435.23}{439.03}\right) = \mathbf{7.54°}$$

5.2.19 Work

Work $w = \frac{U}{g_c}(W_2 \cos \beta_2 + W_3 \cos \beta_3)$

$$= \frac{1236.72}{32.2 * 778}\left[\frac{ft}{s}\right]\left[\frac{lbf.s^2}{lbm.ft}\right]\left[\frac{Btu}{lbf.ft}\right](582.80 \ \sin 41.69 + 1256.86 \ \sin 69.74)\left[\frac{ft}{s}\right]$$

$$= \mathbf{77.83}\left[\frac{\mathbf{Btu}}{\mathbf{lbm.}}\right]$$

5.2.20 Stage Efficiency

Efficiency $\eta = \frac{2wg_c}{C_1^2}$

$$= \frac{2 * 77.83 * 32.2 * 778}{2039.26^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right]$$

$$= \mathbf{93}\%$$

5.2.21 Length of Blade

Length $l = \frac{\dot{m}}{\rho * 2\pi * r_m * C_{x2}} = \frac{72.94}{0.0748 * 2\pi * 1.64 * 435.23}\left[\frac{lbm}{s}\frac{ft^3}{lbm}\frac{1}{ft}\frac{s}{ft}\right] = \mathbf{0.217}\ [\boldsymbol{ft}]$

5.2.22 Spacing of Blade

Assume $b \approx c = 3$ inch (c is chord length)

Spacing $s = \frac{0.85 * b}{2 * (\tan \beta_2 + \tan \beta_3) \cos^2 \beta_3} = \frac{2 * 0.85 * 3}{2 * 12 * (\tan 41.69 + \tan 69.74) \cos^2 69.74} = \mathbf{0.246}\ [\boldsymbol{ft}]$

5.2.23 Number of Blade

Number $N_b = \frac{2\pi r_m}{s} = \frac{2\pi * 1.64}{0.246}\left[\frac{ft}{ft}\right] \approx \mathbf{42}$

5.2.24 Loading Factor

Loading Factor $\psi = \frac{g_c w}{U^2} = \frac{32.2 * 77.83 * 778}{1236.72^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right] = \mathbf{1.27 < 2}$

5.3 1st Stage Design

5.3.1 Specify Δh for the 3rd stage:

Δh for 3rd stage is

We have $\boldsymbol{\Delta h_3} = \mathbf{80} \left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$

5.3.2 Specify Degree of Reaction (R):

Let's specify $\boldsymbol{R_3} = \mathbf{0.34}$

5.3.3 Enthalpy Distribution

Enthalpy in Rotor $\Delta h_r = R_3 * \Delta h_3 = 0.34 * 80 = \mathbf{27.20} \ \left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$

Enthalpy in Stator $\Delta h_s = \Delta h_3 - \Delta h_r = 86.2 - 47.41 = \mathbf{52.80} \left[\frac{\boldsymbol{Btu}}{\boldsymbol{lbm}}\right]$

5.3.4 Adiabatic Velocity

Adiabatic velocity $C_1 = \sqrt{2 * g_c * \Delta h_3}$

$$= \sqrt{2 * 32.2 \left[\frac{lbm.ft}{lbf.s^2}\right] * 80 * 778 \left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{2002.06} \left[\frac{\boldsymbol{ft}}{\boldsymbol{s}}\right]$$

5.3.5 Velocity Ratio

Velocity ratio $\left(\frac{U}{C_1}\right)$ is a function of R. Using Linear approximation.

$$\frac{U}{C_1} \cong \mathbf{0.62}$$

5.3.6 Blade Tangential Velocity

Blade Tangential Velocity $U = 0.62 * C_1 = 0.62 * 2002.06 = \mathbf{1233.23} \left[\frac{\boldsymbol{ft}}{\boldsymbol{s}}\right]$

5.3.7 Speed

Speed $N = 7200\ [RPM] = 7200 * \frac{2\pi}{60} \left[\frac{rad}{s}\right] = \mathbf{753.98} \left[\frac{\boldsymbol{rad}}{\boldsymbol{s}}\right]$

5.3.8 Mean Radius

Root Mean Radius $r_m = \frac{U}{N} = \frac{1233.23}{753.98} \left[\frac{ft}{s}\right] \left[\frac{s}{rad}\right] = \mathbf{1.64}[\boldsymbol{ft}]$

5.3.9 Absolute Velocity at Stator Exit

Absolute Velocity $C_2 = \sqrt{2 * g_c * (1 - R_3) * \Delta h_3}$

$$= \sqrt{2 * 32.2 \left[\frac{lbm.ft}{lbf.s^2}\right] * (1 - 0.34) * 80 * 778 \left[\frac{lbf.ft}{lbm}\right]}$$

$$= \mathbf{1626.48} \left[\frac{\boldsymbol{ft}}{\boldsymbol{s}}\right]$$

5.3.10 Absolute Velocity Angle at Stator Exit

Absolute Velocity Angle α_2 is in the range of (70° - 80°). Let's specify $\alpha_2 = \mathbf{76°}$

5.3.11 Axial Velocity Component at Stator Exit

Axial Velocity Component $C_{x2} = C_2 \cos\alpha_2$

$$= 1626.48 * \cos 76$$

$$= \mathbf{393.48}\ \left[\frac{ft}{s}\right]$$

5.3.12 Relative Velocity at Stator Exit

Relative Velocity $W_2 = \sqrt{C_2^2 + U^2 - 2 * C_2 * U * \sin\alpha_2}$

$$= \sqrt{1626.48^2 + 1233.23^2 - 2 * 1626.48 * 1233.23\ * \sin 76}$$

$$= \mathbf{523.27}\ \left[\frac{ft}{s}\right]$$

5.3.13 Relative Velocity Angle at Stator Exit

Relative Velocity Angle β_2

We have $W_2 \cos\beta_2 = C_{x2} \rightarrow \beta_2 = \cos^{-1}\left(\frac{C_{x2}}{W_2}\right) = \cos^{-1}\left(\frac{393.48}{523.27}\right) = \mathbf{41.24°}$

5.3.14 Relative Velocity Angle at Rotor Exit

Relative Velocity Angle β_3

We have, $R_3 = \frac{C_{x2}}{U}\left(\frac{\tan\beta_3 - \tan\beta_2}{2}\right)$

$$\beta_3 = \tan^{-1}\left(\frac{2UR_3}{C_{x2}} + \tan\beta_2\right) = \tan^{-1}\left(\frac{2 * 1233.23 * 0.34}{393.48} + \tan 41.24\right) = \mathbf{71.61°}$$

5.3.15 Axial Velocity Component at Rotor Exit

Axial Velocity Component $C_{x3} = C_{x2} = \mathbf{393.48}\ \left[\frac{ft}{s}\right]$

5.3.16 Relative Velocity at Rotor Exit

We have, $W_3 \cos\beta_3 = C_{x3}$

Relative Velocity $W_3 = \frac{C_{x3}}{\cos\beta_3}$

$$= \frac{393.48}{\cos 71.61} = \mathbf{1247.23}\ \left[\frac{ft}{s}\right]$$

5.3.17 Absolute Velocity at Rotor Exit

Absolute Velocity $C_3 = \sqrt{W_3^2 + U^2 - 2 * W_3 * U * \sin\beta_3}$

$$= \sqrt{1247.23^2 + 1233.23^2 - 2 * 1247.23 * 1233.23\ * \sin 71.61}$$

$$= \mathbf{396.61}\ \left[\frac{ft}{s}\right]$$

5.3.18 Absolute Velocity Angle at Rotor Exit

Absolute Velocity Angle α_3 is in the range of (0° - 5°)

We have, $C_3 \cos\alpha_3 = C_{x3} \rightarrow \alpha_3 = \cos^{-1}\left(\frac{C_{x3}}{C_3}\right)$

$$= \cos^{-1}\left(\frac{393.48}{396.61}\right) = \mathbf{7.2°}$$

5.3.19 Work

Work $w = \frac{U}{g_c}(W_2 \cos\beta_2 + W_3 \cos\beta_3)$

$$= \frac{1233.23}{32.2 * 778}\left[\frac{ft}{s}\right]\left[\frac{lbf.s^2}{lbm.ft}\right]\left[\frac{Btu}{lbf.ft}\right](523.27\ \sin 41.24 + 1247.23 \sin 71.61)\left[\frac{ft}{s}\right]$$

$$= \mathbf{75.71}\left[\frac{\mathbf{Btu}}{\mathbf{lbm.}}\right]$$

5.3.20 Stage Efficiency

Efficiency $\eta = \frac{2wg_c}{C_1^2}$

$$= \frac{2 * 75.71 * 32.2 * 778}{2002.06^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right]$$

$$= \mathbf{94}\%$$

5.3.21 Length of Blade

Length $l = \frac{\dot{m}}{\rho * 2\pi * r_m * C_{x2}} = \frac{72.94}{0.0748 * 2\pi * 1.64 * 393.48}\left[\frac{lbm}{s}\frac{ft^3}{lbm}\frac{1}{ft}\frac{s}{ft}\right] = \mathbf{0.241}\ [\boldsymbol{ft}]$

5.3.22 Spacing of Blade

Assume $b \approx c = 3$ inch (c is chord length)

Spacing $s = \frac{0.85 * b}{2 * (\tan\beta_2 + \tan\beta_3)\cos^2\beta_3} = \frac{2 * 0.85 * 3}{2 * 12 * (\tan 41.24 + \tan 71.61)\cos^2 71.61} = \mathbf{0.275}\ [\boldsymbol{ft}]$

5.3.23 Number of Blade

Number $N_b = \frac{2\pi r_m}{s} = \frac{2\pi * 1.64}{0.275}\left[\frac{ft}{ft}\right] \approx \mathbf{37}$

5.3.24 Loading Factor

Loading Factor $\psi = \frac{g_c w}{U^2} = \frac{32.2 * 75.71 * 778}{1233.23^2}\left[\frac{lbm.ft}{lbf.s^2}\right]\left[\frac{lbf.ft}{lbm}\right]\left[\frac{s^2}{ft^2}\right] = \mathbf{1.24} < \mathbf{2}$

6. Stage Design Summary

	Symbol	Unit	Stage 1	Stage 2	Stage 3
Delta_h		Btu/lbm.	**86.20**	**83.00**	**80.00**
Reaction (Assumed)	R		**0.30**	**0.32**	**0.34**
Enthalpy in Rotor	h_r	Btu/lbm.	25.86	26.56	27.20
Enthalpy in Stator	h_s	Btu/lbm.	60.34	56.44	52.80
Adiabatic Velocity	C_1	ft./s	2078.20	2039.26	2002.06
Velocity Ratio	U/C_1		0.60	0.61	0.62
Speed	N	RPM	7200.00	7200.00	7200.00
Speed	N	rad/s	753.98	753.98	753.98
Blade Velocity	U	ft./s	1,241.25	1,236.72	1,233.23
Root Mean Radius	r_m	ft.	1.65	1.64	1.64
Velocity angle at Stator Exit (Assumed)	alpha_2	deg.	**73.00**	**75.00**	**76.00**
Velocity at Stator Exit	C_2	ft./s	1,738.74	1,681.61	1,626.48
Axial Velocity Component	C_x2	ft./s	508.36	435.23	393.48
Relative Velocity angle at Stator Exit	Beta_2	deg.	39.66	41.69	41.24
Relative Velocity at Stator Exit	W_2	ft./s	660.39	582.80	523.27
Relative Velocity angle at Rotor Exit	Beta_3	deg.	66.45	69.74	71.61
Relative Velocity at Rotor Exit	W_3	ft./s	1,272.25	1,256.86	1,247.23
Velocity angle at Stator Inlet	alpha_3	deg.	8.39	7.54	7.20
Velocity at Rotor Exit	V_3	ft./s	513.86	439.03	396.61
Work	w	Btu/lbm	78.67	77.83	75.71
Stage Efficiency	eta	%	91%	93%	94%
Length of Blade	l	ft.	0.185	0.217	0.241
Phi (Assumed)			**0.85**	**0.85**	**0.85**
chord length (Assumed)	c	inch	**3.00**	**3.00**	**3.00**
chord length (Assumed)	b	inch	**3.00**	**3.00**	**3.00**
Spacing	s	ft.	0.213	0.246	0.275
Number of blades	N		49	42	37
Loading factor	Ysai		1.27	1.27	1.24

7. Material Selection

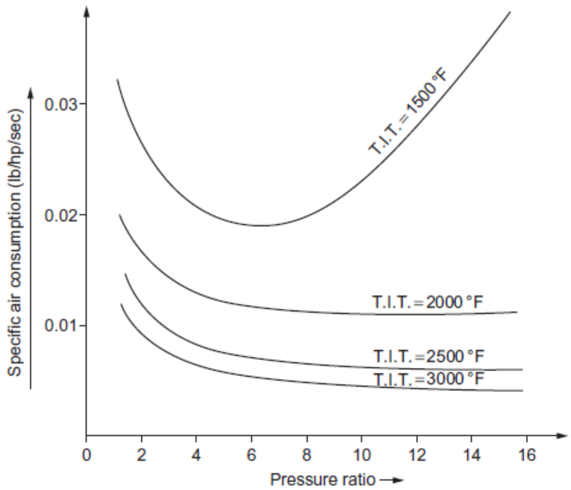


Figure 5 Specific air versus pressure ratio and TIT

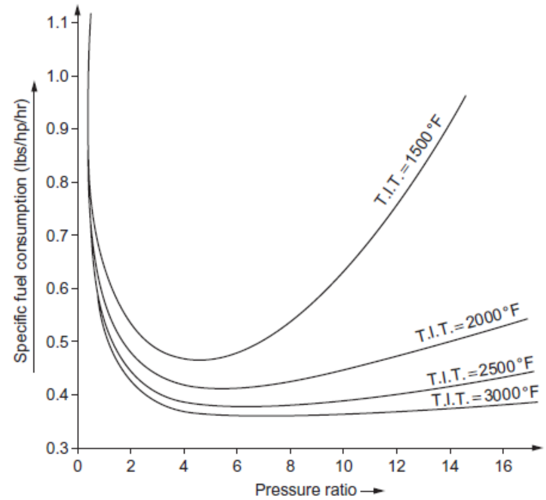


Figure 6 Specific fuel consumption versus pressure ratio and TIT

Temperature limitations are the most crucial limiting factors to gas turbine efficiencies. Figure 5 and 6 show how the increased turbine inlet temperatures decrease both specific fuel and air consumption while increasing efficiency. Materials and alloys that can operate at high temperatures are very costly both to buy and to work on.

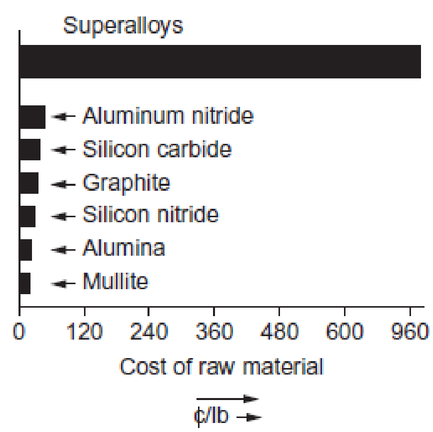


Figure 7 A comparison of raw material costs.

Figure 7 shows relative raw material costs. Thus, the cooling of blades, nozzles, and combustor liners is an integral part of the total materials picture. Since the design of turbomachinery is complex and the efficiency is directly related to material performance, the material selection is of prime importance. Gas turbines exhibit similar problem areas, but these problem areas are of different magnitudes. Turbine components must operate under a variety of stress, temperature, and corrosion conditions. These conditions are more extreme in gas turbine than in steam turbine applications. As a result, the selection of materials for individual components are based on varying criteria in both gas and steam turbines.

Blade life comparison is provided in the form of the stress required for rupture as a function of a parameter that relates time and temperature (the Larson–Miller parameter).

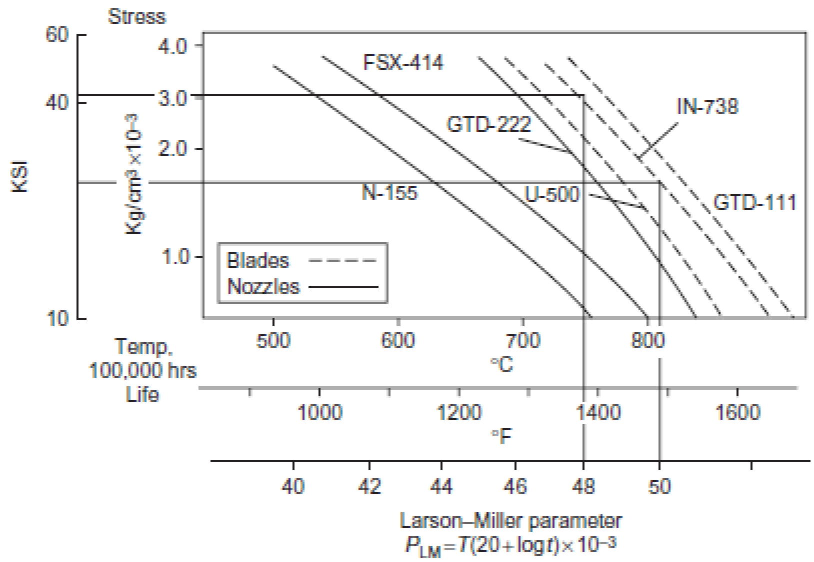


Figure 8 Larson–Miller parameter for various types of blades.

The Larson–Miller parameter is a function of blade metal's temperature and the time the blade is exposed to those temperatures. Figure 8 shows the comparison of some of the alloys used in blade and nozzle applications. This parameter is one of the several important design parameters that must be satisfied to ensure the proper performance of the alloy in a blade application, especially for long service life. Creep life, HCF and LCF, thermal fatigue, tensile strength and ductility, impact strength, hot corrosion and oxidation resistance, producibility, coatability, and physical properties must also be considered.

8. Bearing

The bearings in gas turbines provide support and positioning for the rotating components. Radial support is generally provided by journal, or roller bearings, and axial positioning is provided by thrust bearings. A long service life, a high degree of reliability, and economic efficiency are the chief aims when designing bearing arrangements. To reach these criteria, it is important to examine all the influencing factors:

1. Load and speed
2. Lubrication
3. Temperatures
4. Shaft arrangements
5. Life
6. Mounting and dismounting
7. Noise
8. Environmental conditions

All of the bearings designed for increased stability are obtained at higher manufacturing costs and reduced efficiency. The anti-whirl bearings all impose a parasitic load on the journal, which causes higher power losses to the bearings, and in turn, requires higher oil flow to cool the bearing. Many factors enter into the selection of the proper design for bearings. Some of the factors that affect bearing design follow:

1. Shaft speed range.
2. Maximum shaft misalignment that can be tolerated.
3. Critical speed analysis and the influence of bearing stiffness on this analysis.
4. Loading of the compressor impellers.
5. Oil temperatures and viscosity.
6. Foundation stiffness.
7. Axial movement that can be tolerated.
8. Type of lubrication system and its contamination.
9. Maximum vibration levels that can be tolerated.

Bearing type		Load capacity	Suitable direction of rotation	Tolerance of changing load/speed	Tolerance of misalignment	Space requirement
Plain washer		Poor		Good	Moderate	Compact
Taper land	Bidirectional	Moderate		Poor	Poor	Compact
	Unidirectional	Good		Poor	Poor	Compact
Tilting pad	Bidirectional	Good		Good	Good	Greater
	Unidirectional	Good		Good	Good	Greater

Bearing type	Load capacity	Suitable direction of rotation	Resistance to half-speed whirl	Stiffness and damping
Cylindrical bore	Good		Worst	Moderate
Cylindrical bore with dammed groove	Good			Moderate
Lemon bore	Good			Moderate
Three lobe	Moderate		Increasing	Good
Offset halves	Good			Excellent
Tilting pad	Moderate		Best	Good

10. References

i. Thermo-Fluid System Analysis and Design Professor Dr. Rishi S Raj.

ii. Gas Turbine Engineering Handbook - Meherwan Boyce

iii. Gas Turbine – Claire Soares

iv. Fluid Mechanics and Thermodynamics of Turbomachinery – S.L. Dixon